

Quantum reading of digital memory with non-Gaussian entangled light

J. Prabhu Tej,¹ A. R. Usha Devi,^{1,2,*} and A. K. Rajagopal^{2,3}

¹*Department of Physics, Bangalore University, Bangalore-560 056, India*

²*Inspire Institute Inc., Alexandria, Virginia, 22303, USA.*

³*Harish-Chandra Research Institute, Chhatnag Road, Jhansi, Allahabad 211 019, India.*

It has been shown recently (Phys. Rev. Lett. 106, 090504 (2011)) that entangled light with Einstein-Podolsky-Rosen (EPR) correlations retrieves information from digital memory better than any classical light. In identifying this, a model of digital memory with each cell consisting of reflecting medium with two reflectivities (each memory cell encoding the binary numbers 0 or 1) is employed. The readout of binary memory essentially corresponds to discrimination of two Bosonic attenuator channels characterized by different reflectivities. The model requires an entire mathematical paraphernalia of continuous variable Gaussian setting for its analysis, when arbitrary values of reflectivities $0 \leq r_0, r_1 \leq 1$ are considered. Here we restrict to a basic quantum read-out mechanism with non-Gaussian entangled states of light, with the binary channels to be discriminated being (i) ideal memory characterized by reflectivity $r_1 = 1$ (identity channel) and (ii) a thermal noise channel – where the signal light illuminating the memory location gets completely lost ($r_0 = 0$) and only a white thermal noise hitting the upper side of the memory reaches the decoder. We compare the quantum reading efficiency of entangled light with any classical source of light in this model. We show that entangled transmitters offer better reading performance than any classical transmitters of light in the regime of low signal intensity.

I. INTRODUCTION

Entangled states are known to offer enhanced performance sensitivity over unentangled ones in quantum channel discrimination [1, 2]. This opens up several potential implications in quantum information protocols. Formulating target detection as a channel discrimination problem, advantage of entangled light transmitter over a classical source of light has been explored in Refs. [3–8], where light received from a far target region is used to ascertain if a low reflectivity object – immersed in a bright thermal noise – is present or not. In a different regime, EPR correlated light is shown to offer remarkable improvement in the read-out of information from a digital memory over any classical light [9]. Here, a binary memory system is modelled as an array of cells, where each memory cell is composed of a reflecting medium with two possible reflectivities to store a bit of information. The task of digital readout in this model is essentially to discriminate the two channels characterized by beam splitters with reflectivities r_0, r_1 , by distinguishing the states of reflected light. In the basic scheme of reading digital memory, a transmitter emits a global state of light, composed of M copies of signal (with N_S mean number of photons per signal mode) and L copies of idler. The signal light illuminates the memory cell of optical reflectivity r_0 (encoding bit value 0) or r_1 (encoding bit value 1) and the reflected light is detected along with the idler at the receiver. A optimal measurement scheme is employed to decode the bit value 0 or 1 with an error probability P_e . For a fixed total mean number $N = M N_S$ of signal photons illuminating each memory cell, it was shown that

a non-classical transmitter emitting EPR correlated light can retrieve more information than any classical source of light, in the regime of few photons ($N < 10^2$) and for typical optical memories with high reflectivities [9]. Following Ref. [9], the use of non-classical transmitters of light to read digital memory has been referred to as *quantum reading*. Several other supplementary features of quantum reading have been explored recently [10–16].

In this paper we consider a simple model of digital memory consisting of cells of reflectivity $r_1 = 1$, encoding bit value 1 (corresponds to the situation where signal light is reflected without any loss) and $r_0 = 0$, encoding bit value 0 (corresponds to total loss of signal light, while white thermal noise is received by the detector). The strategy of readout thus reduces to the discrimination of identity and thermal channels. This scheme offers a simple analysis to obtain analytical results for various bounds on error probability. Further, it offers a simplified approach to explore the quantum advantage of non-Gaussian entangled light vs any classical light in digital memory reading. Here, we study the quantum reading efficiency of (i) a class of path entangled bipartite states of photons (known popularly as $\mathcal{M}\&\mathcal{M}$ states), proposed in Ref. [17] in the context of robust quantum optical metrology and (ii) entangled states of light produced by combining a single photon with a coherent state in a 50:50 beam-splitter [18]. We carryout a comparison of quantum-classical read-out performances under a *local energy constraint* [16] i.e., by fixing the mean number N_S of photons per signal mode illuminating the memory cell and the number M of signal modes – in contrast to the analysis with a *global energy constraint* [9], where the total average number of photons $N = M N_S$ shining the memory is held fixed.

We organize the paper as follows: In Sec. II we outline preliminary concepts of quantum reading in general. We

*Electronic address: arutth@rediffmail.com

describe our basic model of memory where bit value 1 is encoded in a cell consisting of a perfect mirror of reflectivity $r_1 = 1$ – returning the light to the receiver without any loss – and bit value zero is encoded in a cell of reflectivity zero – sending thermal light to the receiver. We illustrate in Sec. III, the quantum advantage in the readout of classical information stored in the digital memory, which can be achieved by using non-Gaussian entangled light, in the regime of low signal intensity. Section IV has concluding remarks.

II. BASIC MODEL OF READING DIGITAL OPTICAL MEMORY

In the model proposed by Pirandola [9], storage of binary data 0, 1 in a digital memory corresponds to encoding them in channels (cells of the memory) \mathcal{E}_0 , \mathcal{E}_1 , which are beam splitters of reflectivities r_0 and r_1 respectively. Readout is a process of channel decoding, which corresponds to discriminating the channels. This is carried out by sending input light to illuminate the cells and then distinguishing the reflected output states of light with the help of suitable measurements at the receiving end.

Let us consider an input Bosonic density matrix $[\rho_{\text{in}}]^{\otimes(M,M')}$ consisting of M copies of signal (S) and M' copies of idler (I), each copy of the signal mode carrying mean photon number N_S . The output state of light $[\rho_{\text{out}}^{(u)}]^{\otimes(M,M')}$ received at the detector is a combined state of M signal modes reflected after illuminating a cell of reflectivity r_u , $u = 0, 1$ and M' idler modes:

$$[\rho_{\text{out}}^{(u)}]^{\otimes(M,M')} = \left(\mathcal{E}_u^{\otimes M} \otimes \mathcal{I}^{\otimes M'} \right) [\rho_{\text{in}}^{\otimes(M,M')}] , \quad (1)$$

where the channel $\mathcal{E}_u^{\otimes M}$ acts on M signal modes and the identity channel $\mathcal{I}^{\otimes M'}$ operates on M' idler modes.

Minimum error probability in discriminating the two states $[\rho_{\text{out}}^{(u)}]^{\otimes(M,M')}$, $u = 0, 1$ for a given input state $[\rho_{\text{in}}]^{\otimes(M,M')}$ is given by [19, 20]

$$P_{\text{err}} = \frac{1}{2} \left[1 - \frac{1}{2} \left\| [\rho_{\text{out}}^{(0)}]^{\otimes(M,M')} - [\rho_{\text{out}}^{(1)}]^{\otimes(M,M')} \right\| \right], \quad (2)$$

with $\|A\| = \text{Tr}[\sqrt{A^\dagger A}]$ denoting the tracenorm of A , which is the sum of absolute eigenvalues of A .

Average information, in bits, retrieved from digital memory is quantified by [9]

$$J = 1 - H(P_{\text{err}}) \quad (3)$$

where $H(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ is the binary Shannon entropy.

Finding the eigenvalues of $[\rho_{\text{out}}^{(0)}]^{\otimes(M,M')} - [\rho_{\text{out}}^{(1)}]^{\otimes(M,M')}$, in order to evaluate the error probability (2), is a hard computational problem. However, several easier-to-compute upper and lower bounds on error probability are found useful [21, 22]. Of interest is the quantum Chernoff bound [22], which gives the asymptotic rate exponent of error probability with M signal modes as,

$$P_{\text{err}} \leq P_{\text{err,QCB}} := \frac{1}{2} \left(\min_{0 \leq s \leq 1} \text{Tr}\{[\rho_{\text{out}}^{(0)}]^s [\rho_{\text{out}}^{(1)}]^{1-s}\} \right)^M \quad (4)$$

To explore the advantage of non-classical light over any classical light, one needs to check if the probability of error (2) with any non-classical light is smaller than that when classical light is employed in the readout process.

Any Bosonic density matrix $[\rho^{(\text{cl})}]^{\otimes(M,M')}$ of electromagnetic radiation (with M signal modes and M' idler modes) is classical, when its decomposition in terms of coherent states is positive [23, 24]:

$$[\rho^{(\text{cl})}]^{\otimes(M,M')} = \int d^2\alpha_1 \dots \int d^2\alpha_M \int d^2\alpha'_1 \dots \int d^2\alpha'_{M'} P(\alpha_1, \dots, \alpha_M; \alpha'_1, \dots, \alpha'_{M'}) \otimes_{k=1}^M |\alpha_k\rangle\langle\alpha_k| \otimes_{l=1}^{M'} |\alpha'_l\rangle\langle\alpha'_l|, \quad (5)$$

where $\{|\alpha_k\rangle, (|\alpha'_l\rangle)\}$ denote coherent states of the signal (idler) modes; the function $P(\alpha_1, \dots, \alpha_M; \alpha'_1, \dots, \alpha'_{M'})$ is a legitimate probability distribution, being positive and normalized to unity. It was shown in Ref. [9] that probability of error with any classical state $[\rho^{(\text{cl})}]^{\otimes(M,M')}$ characterized by a positive \mathcal{P} -representation is lower bounded by a quantity $C(M, N_S, r_0, r_1)$, which depends on the number of signal modes M , average signal intensity per

mode N_S , and the reflectivities r_0, r_1 as,

$$P_{\text{err}}^{\text{C}} \geq C(M, N_S, r_0, r_1) := \frac{1 - \sqrt{1 - e^{-M N_S (\sqrt{r_1} - \sqrt{r_0})^2}}}{2}. \quad (6)$$

It follows that the *maximum* information decoded from digital memory (of reflectivities r_0, r_1), by employing any classical source of light (with mean intensity $N = M N_S$ shining each cell) is given by,

$$J_{\text{max,C}} = 1 - H[C(M, N_S, r_0, r_1)]. \quad (7)$$

On the other hand, employing a non-classical source of

light of mean intensity $N = M N_S$, it is possible to retrieve at least

$$J_{\min, \mathcal{Q}} = 1 - H(P_{\text{err}, \mathcal{QCB}}) \quad (8)$$

bits of average information. Improvement of non-classical source of light over any classical transmitter in the readout of information from digital memory is registered if the *minimum information gain* [9]

$$G(M, N_S, r_0, r_1) = J_{\min, \mathcal{Q}} - J_{\max, \mathcal{C}} \quad (9)$$

is positive. It was shown in Ref. [9] that in the regime of few photons ($N < 10^2$) and high reflectivities, non-classical EPR correlated light is capable of retrieving more information (via positive gain G) than any classical light of same average intensity $N = M N_S$. In particular, with small number of signal photons employed to decode *ideal memories* $r_1 = 1$ and $0 \leq r_0 \leq 1$, it was identified [9] that the value of G can be remarkably large.

The quantum effects ($G > 0$) persist even when stray thermal background photons (with $N_B \approx 10^{-2} - 10^{-5}$ average number of noise photons per mode) hit the rear side of the memory and reach the receiver along with the reflected signal light [9, 16]. In this case, the minimum error probability in decoding the memory by any classical light is lower bounded [16] by a quantity $\mathcal{C}(M, N_S, N_B, r_0, r_1)$ as

$$P_{\text{err}, \mathcal{C}} \geq \mathcal{C}(M, N_S, N_B, r_0, r_1) \\ := \frac{1 - \sqrt{1 - \mathcal{F}^M(N_S, N_B, r_0, r_1)}}{2} \quad (10)$$

where

$$\mathcal{F}(N_S, N_B, r_0, r_1) = \frac{\exp\left[-\frac{(\sqrt{r_0} - \sqrt{r_1})^2}{\gamma} N_S\right]}{\sqrt{\gamma^2 + \theta} - \sqrt{\theta}}, \\ \gamma = 1 + (2 - r_0 - r_1) N_B, \\ \theta = 4 N_B^2 \prod_{u=0,1} (1 - r_u)[1 + (1 - r_u) N_B]. \quad (11)$$

Substituting $\mathcal{C}(M, N_S, N_B, r_0, r_1)$ in (7), and re-expressing (9), it is found that for given values of M, N_S, N_B, r_0, r_1 , a non-classical transmitter emitting EPR correlated light can be used to beat the classical readout (verified via $G(M, N_S, N_B, r_0, r_1) > 0$) even in the presence of thermal noise [9, 16]. In Ref. [9] the outperformance of non-classical transmitter over classical one was analyzed under a *global energy constraint*, where total average number of photons $N = M N_S$ illuminating each memory cell is held fixed. On the other

hand, a different kind of quantum-classical comparison is performed based on *local energy constraint* i.e., by fixing the mean number of signal photons N_S in each mode and the number of signal modes M .

Here, we confine ourselves to the case of *ideal memory* with optical reflectivity $r_1 = 1$ (\mathcal{E}_1 being the identity channel) and $r_0 = 0$, which corresponds to complete loss of signal photons illuminating the cell, while only stray thermal photons hitting the upper side of the memory reach the detector (i.e., \mathcal{E}_0 is a thermal channel). This basic model of memory is useful to explore reading abilities of some interesting classes of entangled non-Gaussian states of light in comparison with classical light.

III. QUANTUM READING PERFORMANCE OF ENTANGLED NON-GAUSSIAN STATES OF LIGHT

In this section we explore the quantum efficiency in reading a digital memory with reflectivities $r_1 = 1, r_0 = 0$ and thermal noise N_B , by employing two different families of non-Gaussian entangled states.

A. Path entangled $\mathcal{M}\&\mathcal{M}$ photon states

Huver et. al. [17] proposed a class of bipartite path entangled photon Fock states and showed that they perform better than N00N states in the limit of practical quantum optical metrology with appreciable photon loss. The family of two mode photon states, referred to as $\mathcal{M}\&\mathcal{M}$ states, are defined as follows:

$$|m :: m'\rangle = \frac{1}{\sqrt{2}} [|m, m'\rangle + |m', m\rangle], \quad (12)$$

where $m, m' = 0, 1, 2, \dots$ and $m > m'$. When $m' = 0$, the states reduce to the family of N00N states.

Our focus here is to study the reading performance of this family of photon states in comparison with any classical light. Let us consider M copies of the state with the first mode being signal (S) and the second being idler (I) mode. The average intensity in each signal mode is given by [25] $N_S = \langle a_S^\dagger a_S \rangle = (m + m')/2$. With M identical copies of the state (12) as input, the corresponding output states of the beam splitter channels \mathcal{E}_u , $u = 0, 1$, characterized by reflectivities $r_0 = 0, r_1 = 1$ and thermal bath with N_B noise photons are given by,

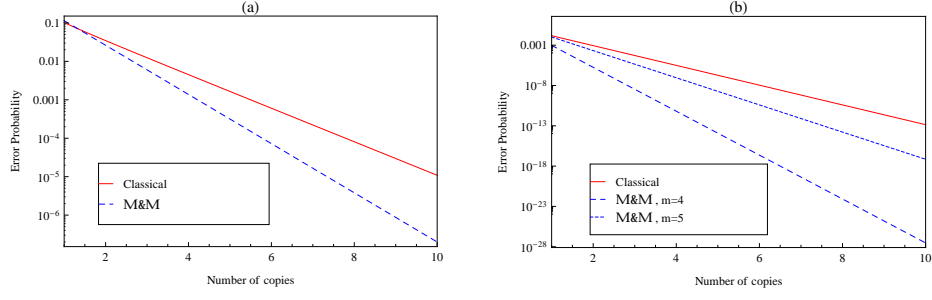


FIG. 1: Quantum Chernoff bound on error $P_{\text{err,QCB}}^{(\mathcal{M}\&\mathcal{M})}(M, N_S, N_B, m)$ of a $\mathcal{M}\&\mathcal{M}$ family of states $\{|m :: 2N_S - m\rangle, N_S < m < 2N_S\}$ and the classical error bound $\mathcal{C}(M, N_S, N_B)$ vs number of copies M for the values (a) $N_S = 1, m = 2$ (b) $N_S = 3, m = 4, m = 5$ and thermal noise $N_B = 10^{-1}$.

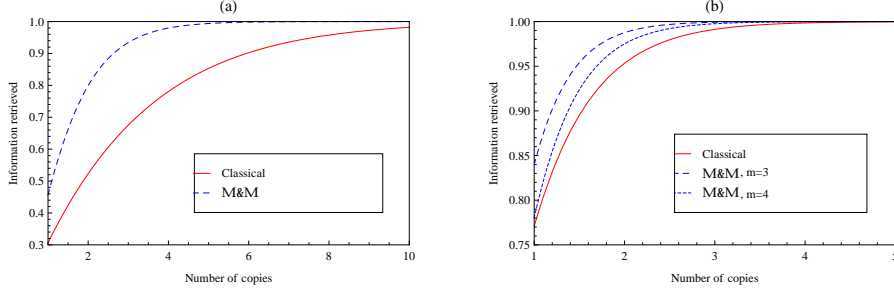


FIG. 2: Maximum information retrieved $J_{\text{max},c}$ in the readout process from a classical transmitter and the minimum information decoded from quantum light of $\mathcal{M}\&\mathcal{M}$ state for (a) $N_S = 0.5$ and $N_B = 10^{-5}$ (b) $N_S = 2.5$ and $N_B = 1$ with $m = 3$ and $m = 4$ plotted as a function of number of copies M .

$$[\rho_{\text{out}}^{(0)}]^{\otimes(M,M)} = [|m :: m'\rangle_{SI}\langle m :: m'|]^{\otimes(M,M)} \quad (13)$$

$$\begin{aligned} [\rho_{\text{out}}^{(1)}]^{\otimes(M,M)} &= [\rho_{\text{Th}}(N_B)]^{\otimes M} \otimes \{\text{Tr}_S[|m :: m'\rangle_{SI}\langle m :: m'|]\}^{\otimes M} \\ &= [\rho_{\text{Th}}(N_B)]^{\otimes M} \otimes \left\{ \frac{1}{2} [|m\rangle_I\langle m| + |m'\rangle_I\langle m'|] \right\}^{\otimes M}, \end{aligned} \quad (14)$$

where $\rho_{\text{Th}}(N_B) = \sum_{n=0}^{\infty} \frac{N_B^n}{(N_B + 1)^{n+1}} |n\rangle\langle n|$ denotes thermal state of N_B average photons.

As one of the states to be discriminated is pure, the

Chernoff error bound $P_{\text{err,QCB}}$ (see (4)) for discriminating the output states (13) and (14) can be readily evaluated (as optimum value of s in (4) is equal to one when one of the states is pure):

$$\begin{aligned} P_{\text{err,QCB}}^{(\mathcal{M}\&\mathcal{M})}(M, N_S, N_B, m) &= \frac{1}{2} \left({}_{SI}\langle m :: m'| \left[\rho_{\text{Th}}(N_B) \right] \otimes \left\{ \frac{1}{2} [|m\rangle_I\langle m| + |m'\rangle_I\langle m'|] \right\} |m :: m'\rangle_{SI} \right)^M \\ &= \frac{1}{2} \left(\frac{N_B^{m'}}{4(N_B + 1)^{m'+1}} \left[\left(\frac{N_B + 1}{N_B} \right)^m + \left(\frac{N_B + 1}{N_B} \right)^{-(m'+1)} \right] \right)^M, \quad N_S = \frac{m + m'}{2}. \end{aligned} \quad (15)$$

Improved reading efficiency of the $\mathcal{M}\&\mathcal{M}$ family of states $\{|m :: 2N_S - m\rangle, N_S < m < 2N_S\}$ over any clas-

sical transmitter of the same signal profile $\{M, N_S\}$ is ensured if the Chernoff bound on error probability (15) – which affects the readout – is smaller than the lower bound (10) on classical error probability itself i.e., $P_{\text{err, QCB}}^{(\mathcal{M}\&\mathcal{M})}(M, N_S, N_B, m) < \mathcal{C}(M, N_S, N_B)$. In Fig. 1 we plot the bounds $P_{\text{err, QCB}}^{(\mathcal{M}\&\mathcal{M})}(M, N_S, N_B, m)$, $\mathcal{C}(M, N_S, N_B)$ on error probabilities in logarithmic scale vs the number of copies M , for fixed signal energy (a) $N_S = 1, m = 2$ and (b) $N_S = 3, m = 4, 5$ respectively, with a thermal noise of $N_B = 10^{-1}$ photons. We identify that the Chernoff bound on error $P_{\text{err, QCB}}^{(\mathcal{M}\&\mathcal{M})}(M, N_S, N_B)$ of $\mathcal{M}\&\mathcal{M}$ transmitter is smaller than the lower bound on error probability $\mathcal{C}(M, N_S, N_B)$ on all classical transmitters, even though both quantum and classical error probability bounds approach zero very sharply with few copies M – indicating efficient reading possibilities in both situations, for memory cell reflectivities $r_1 = 1, r_0 = 0$. This feature is also reflected in Fig. 2, where we have plotted the maximum information $J_{\text{max}, \mathcal{C}}$, retrieved in the classical scenario and minimum information $J_{\text{min}, \mathcal{Q}}$ decoded with quantum $\mathcal{M}\&\mathcal{M}$ transmitter in the readout for (a) $N_S = 0.5, m = 1$ and $N_B = 10^{-5}$, (b) $N_S = 2.5, m = 3, 4$ and $N_B = 1$. While quantum advantage is evident as $J_{\text{max}, \mathcal{C}} < J_{\text{min}, \mathcal{Q}}$ with a single copy itself, it is seen that both $J_{\text{max}, \mathcal{C}}, J_{\text{min}, \mathcal{Q}} \rightarrow 1$ with very few number of copies – implying perfect readout by both classical and quantum transmitters of light. A positive information gain $G \approx 0.3$ can be achieved for low intensity signals $N_S \leq 1$, even with a single copy of the $\mathcal{M}\&\mathcal{M}$ state.

B. Entangled light produced by combining a single photon with coherent light in a 50:50 beam-splitter

It is wellknown that the output Bosonic state from a 50:50 beam splitter is entangled if one of the ports has non-classical state as its input [26–28]. Robustness of entanglement of the non-Gaussian state of light created by combining a single photon with coherent light in a beam splitter is analyzed very recently [18] at various scales

by tuning the intensity of the coherent light. We carry out the classical-quantum comparison by studying the reading performance of this family of entangled states.

With a single photon state $|1\rangle$ and coherent state $|\alpha\rangle$ of intensity $|\alpha|^2$ are sent through a 50:50 beam splitter, the output state (one of the output modes is denoted as signal and the other as idler) is given by,

$$|\Psi_{SI}\rangle = U_{\text{BS}} [a_S^\dagger |0\rangle_S \otimes D_I(\alpha) |0\rangle_I] \quad (16)$$

$$= [D_S(\alpha/\sqrt{2}) \otimes D_I(\alpha/\sqrt{2})] \left[\frac{|1\rangle_S \otimes |0\rangle_I + |0\rangle_S \otimes |1\rangle_I}{\sqrt{2}} \right].$$

Here, the displacement operator $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$ generates the coherent state $|\alpha\rangle = D(\alpha)|0\rangle$; a_S, a_I denote the annihilation operators of the signal and idler modes and the 50:50 beam splitter leads to the transformation $(a_S, a_I) \xrightarrow{U_{\text{BS}}} ((a_S - a_I)/\sqrt{2}, (a_S + a_I)/\sqrt{2})$. The average intensity in the signal mode of the state (16) is given by $N_S = \langle a_S^\dagger a_S \rangle = (|\alpha|^2 + 1)/2$.

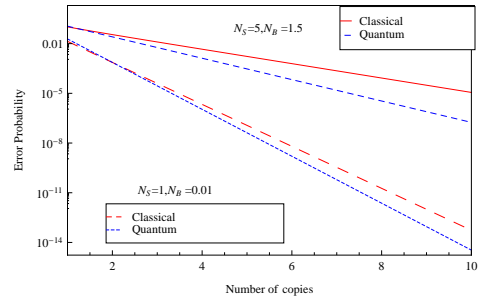


FIG. 3: Error probability bounds $P_{\text{err, QCB}}^{(\Psi)}(M, N_S, N_B)$ and $\mathcal{C}(M, N_S, N_B)$ as a function of number of copies M for different choices of mean signal intensity $N_S = 5, 1$; thermal noise $N_B = 1.5, 0.01$.

Considering M identical copies of the state (16), and sending the signal mode to shine the memory cells, the states of the reflected light combined with the idler modes are given by,

$$[\rho_{\text{out}}^{(0)}]^{\otimes(M, M)} = [|\Psi_{SI}\rangle\langle\Psi_{SI}|]^{\otimes(M, M)} \quad (17)$$

$$[\rho_{\text{out}}^{(1)}]^{\otimes(M, M)} = [\rho_{\text{Th}}(N_B)]^{\otimes M} \otimes \{\text{Tr}_S[|\Psi\rangle_{SI}\langle\Psi_{SI}|]\}^{\otimes M}$$

$$= [\rho_{\text{Th}}(N_B)]^{\otimes M} \otimes \left\{ D_I\left(\frac{\alpha}{\sqrt{2}}\right) \left[\frac{1}{2} (|0\rangle_I\langle 0| + |1\rangle_I\langle 1|) \right] D_I^\dagger\left(\frac{\alpha}{\sqrt{2}}\right) \right\}^{\otimes M}, \quad (18)$$

The Chernoff upper bound on the error probability $P_{\text{err, QCB}}$ affecting the decoding of the binary memory

(by discriminating the output states (17) and (18)) is obtained, after simplifications as,

$$\begin{aligned}
P_{\text{err,QCB}}^{(\Psi)}(M, N_S, N_B) &= \frac{1}{2} \left(\langle \Psi_{SI} | \left[\rho_{\text{Th}}(N_B) \right] \otimes \left\{ D_I \left(\frac{\alpha}{\sqrt{2}} \right) \left[\frac{1}{2} (|0\rangle_I \langle 0| + |1\rangle_I \langle 1|) \right] D_I^\dagger \left(\frac{\alpha}{\sqrt{2}} \right) \right\} | \Psi_{SI} \rangle \right)^M \\
&= \frac{1}{2} \frac{e^{-\frac{M(2N_S-1)}{2(N_B+1)}}}{[4(N_B+1)]^M} \left[1 + \frac{(2N_S-1)(1+N_B+N_B^2)}{2(N_B+1)^2} \right]^M
\end{aligned} \tag{19}$$

We now carry out the quantum-classical comparison in reading efficiency of the entangled light $|\Psi_{SI}\rangle$ over any classical radiation of the same signal profile $\{M, N_S\}$. In Fig. 3 it is displayed that the Chernoff bound on error probability (19) is smaller than the lower bound (10) on classical error probability (in logarithmic scale), for some typical values of low signal intensities $N_S = 1, 5$, and thermal noise of $N_B = 10^{-2}, 1.5$ photons respectively. In Fig. 4, we have plotted the maximum decoded information $J_{\text{max,C}}$, when the readout process employs classical light and minimum information $J_{\text{min,Q}}$ that can be retrieved using quantum state (16) of light. We find that both the informations $J_{\text{max,C}}, J_{\text{min,Q}} \rightarrow 1$ with small number of copies M indicating that with ideal memory $r_1 = 1$ encoding bit value 1 and thermal noise mixed with perfectly transmitting memory $r_0 = 0$ encoding the bit value 0, the reading efficiency of both classical and quantum light is significantly large. However, there is a quantum advantage and a positive gain of around $G \approx 0.3$ can be realized for average signal photons $0.5 \leq N_S \leq 1$ with even one copy of the state (16).

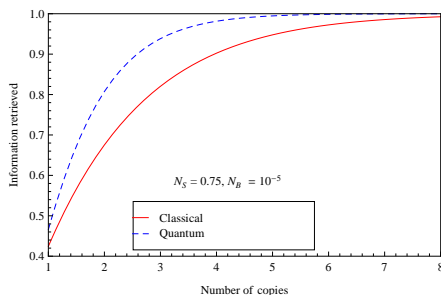


FIG. 4: Maximum information $J_{\text{max,C}}$ retrieved from digital memory, when classical light is used and minimum information $J_{\text{min,Q}}$ decoded by employing quantum entangled state (16) of light as a function of number of copies, M for the entangled state for $N_S = 0.75$ and $N_B = 10^{-5}$.

IV. CONCLUSION

In conclusion, following the *quantum reading* scheme of digital memory motivated in Ref. [9], we have explored a model of memory consisting of perfectly reflecting mirror (reflectivity $r_1 = 1$), encoding bit value 1, and a completely transmitting cell of reflectivity $r_0 = 0$ (with stray thermal photons hitting the upper side of the memory received by the detector) encoding bit value 0. This simple scheme is useful to evaluate the error rate affecting the readout of memory with pure entangled states of light. We have explored the quantum reading efficiency of (i) $\mathcal{M}\&\mathcal{M}$ class of path entangled photons [17] and (ii) entangled states of light produced by combining a single photon with a coherent state in a 50:50 beam-splitter [18] by comparing them with any classical transmitter of light for fixed signal profiles, under *local energy constraint* [16]. It is identified that even a single copy of faint non-Gaussian entangled light (with average signal intensity $N_S \approx 1$) can offer improvement (with a maximum of 30% information gain) in the readout of binary memory than any classical light. The results obtained here agree in general (when confined to specific values of reflectivities $r_1 = 1, r_0 = 0$) with the identifications of Ref. [16], where quantum-classical comparison is done by employing entangled EPR correlated light. The improved reading efficiency recognised with different sources of entangled light over classical light in the low signal intensity regime has implications towards short read-out time (or high data transfer rate), ability for dense storage [9, 16]. Consequently, enhancement of readout in the faint light limit – established with different sources of quantum light – strengthens technological possibilities of decoding photo degradable organic memories.

- [1] M. F. Sacchi, Phys. Rev. A, **71** 062340 (2005); Phys. Rev. A, **72**, 014305 (2005).
- [2] M. Piani, J. Watrous, Phys. Rev. Lett. **102**, 250501 (2009).
- [3] S.-H. Tan, B. I. Erkmen, V. Giovannetti, S. Guha, S. Lloyd, L. Maccone, S. Pirandola, and J. H. Shapiro, Phys. Rev. Lett. **101**, 253601 (2008).
- [4] S. Lloyd, Science **321**, 1463 (2008).

- [5] J. H. Shapiro and S. Lloyd, New J. Phys. **11**, 063045 (2009).
- [6] S. Guha and B. I. Erkmen, Phys. Rev. A **80**, 052310 (2009).
- [7] A. R. Usha Devi and A. K. Rajagopal, Phys. Rev. A **79**, 062320 (2009).
- [8] H. P. Yuen and R. Nair, Phys. Rev. A **80**, 023816 (2009).
- [9] S. Pirandola, Phys. Rev. Lett. **106**, 090504 (2011).

- [10] R. Nair, Phys. Rev. A **84**, 032312 (2011).
- [11] S. Pirandola, C. Lupo, V. Giovannetti, S. Mancini, and S. L. Braunstein, New J. Phys. **13**, 113012 (2011).
- [12] O. Hirota, arXiv:1108.4163.
- [13] S. Guha, Z. Dutton, R. Nair, J. Shapiro, and B. Yen, Information Capacity of Quantum Reading, in Laser Science, OSA Technical Digest, Paper LTuF2 (Optical Society of America, 2011).
- [14] M. M. Wilde, S. Guha, S.-H. Tan, and S. Lloyd, arXiv:1202.0518.
- [15] M. Dall'Arno, A. Bisio, G. M. D'Ariano, M. Miková, M. Ježek, and M. Dušek, Phys. Rev. A **85**, 012308 (2012).
- [16] G. Spedalieri, C. Lupo, S. Mancini, S. L. Braunstein and S. Pirandola, Phys. Rev. A **86**, 012315 (2012).
- [17] S. D. Huver, C. F. Wildfeuer, and J. P. Dowling, Phys. Rev. A **78**, 063828 (2008).
- [18] P. Sekatski, N. Sangouard, M. Stobińska, F. Bussières, M. Afzelius, and N. Gisin, arXiv: 1206.1870.
- [19] C W Helstrom, *Quantum Detection and Estimation Theory*, (Academic Press, New York, 1976).
- [20] A. S. Holevo, Theory Probab. Appl. **23**, 411 (1979).
- [21] V. Kargin, Ann. Stat. **33**, 959 (2005).
- [22] K. M. R. Audenaert et. al., Phys. Rev. Lett. **98**, 160501 (2007); J. Calsamiglia et. al, Phys. Rev. A **77**, 032311 (2008).
- [23] E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963).
- [24] R. J. Glauber, Phys. Rev. **131**, 2766 (1963).
- [25] It is convenient to express the family of $\mathcal{M}\&\mathcal{M}$ states as $|m :: m'\rangle \equiv |m :: 2N_S - m\rangle$, $N_S < m \leq 2N_S$ in terms of the average signal intensity $N_S = (m + m')/2$.
- [26] X. B. Wang, Phys. Rev. A **66**, 024303 (2002).
- [27] J. K. Asboth, J. Calsamiglia, H. Ritsch, Phys. Rev. Lett. **94**, 173602 (2005).
- [28] J. Solomon Ivan, N. Mukunda, and R. Simon, arxiv:0603255.